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Friction drag resulting from the simultaneous imposed motions of a freestream and its bounding surface

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Abstract

The problem of laminar fluid flow which results from the simultaneous motions of a freestream and its bounding surface in the same direction has been investigated numerically. A special focus was to establish thresholds demarking the degree of interaction between the two imposed motions. For the case in which the freestream velocity, U_∞, is greater than the surface velocity, U_S, it was found that surface velocities as great as $0.183U_\infty$ could be tolerated without causing an appreciable effect on the drag force. On the other hand, in the case where the surface velocity exceeds the freestream velocity, the drag is significantly affected by freestream velocities $\geqslant 0.0686U_S$.

The validity of the so-called *relative-velocity model* was investigated. In that model, the relative velocity between the two media is used in conjunction with the drag-force formula for the dominant of the two motions acting alone. The results of exact solutions demonstrate that this model is flawed and underpredicts the drag force. Finally, a new model, termed the *similarity-based, relative-velocity model* was devised by which exact similarity solutions were obtained. The drag results provided by this model may be considered to be the definitive information for application.

Keywords: Moving surface; Moving fluid; Drag forces; Laminar flow; Exact similarity solutions; Web processing

1. Introduction

In the processing of web-like materials such as polymer sheets, paper, linoleum, or roofing shingles, one of the commonly encountered approaches is to cure and/or dry the moving web by the use of a parallel gas flow. A schematic diagram of the process setup is presented in Fig. 1. As seen there, the material to be processed is supplied from a roll situated at the upstream end of the processing station. The processed product is collected by a take-up roll located at the downstream end of the station. The gas flow is supplied by means of a converging nozzle, the degree of convergence being such that the boundary layer thickness of the gas flow at the nozzle exit is negligible. The flow emerges from the nozzle as a straight, parallel freestream with uniform velocity. The relative motion between the unrolling web and the edge of the nozzle is accommodated by brushes. A sideview of the physical situation illustrated in Fig. 1 is displayed in Fig. 2.

In the textbook literature (Bejan, 1993; Cengel, 2003; Incropera and DeWitt, 2002; Kaviany, 2002; Kreith and Bohn, 2001; Rohsenow and Choi, 1961; Suryanarayana, 1995), it appears that no distinction has been drawn between the fluid flow adjacent to a moving web such as that pictured in Figs. 1 and 2 and the classical stationary flat-plate boundary layer problem, often termed the Blasius problem (Blasius, 1908). The conclusion that can be drawn from the aforementioned textbooks is that those authors have assumed that when a moving fluid passes

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Nomenclature

$f(\xi)$	similarity function based on the relative velocity between the fluid and the bounding surface
F	drag force
F_{exact}	drag force based on a similarity solution
$F^{U^{I}_{rel}}$	drag force based on relative-velocity model for $U_\infty > U_S$
$F^{U^{II}_{rel}}$	drag force based on relative-velocity model for $U_S > U_\infty$
L	streamwise length of the bounding surface
u, v	velocity components in the x and y directions, respectively
U_∞	freestream velocity
U_S	velocity of the moving surface
U_{rel}	relative velocity between the moving surface and the freestream
W	width of bounding surface normal to the direction of motion
x, y	horizontal and vertical coordinates

Greeks

α	velocity ratio, U_∞/U_S
μ	dynamic viscosity of fluid
ν	kinematic viscosity of fluid
ψ	streamfunction
ξ	similarity variable based on the relative velocity, U_{rel}

Superscripts

I	corresponds to the case for which $U_\infty > U_S$
II	corresponds to the case for which $U_S > U_\infty$

over a stationary solid and/or when a moving solid passes through a stationary fluid, it is only the *relative velocity* between the surface and the fluid that determines the fluid flow and heat transfer characteristics. This model will hereafter be termed the *relative-velocity model*. For the determination of the drag force, this model uses the magnitude of the relative velocity in conjunction with the drag formula for the case in which only one of the participating media is in motion.

The problem of a surface moving through a stationary fluid was first analyzed by Sakiadis (1961) and experimentally investigated by Tsou et al. (1967). Subsequently, the situation in which both the surface and the fluid are in motion in the same direction has been analyzed by Abdelhafez (1985) and Chen (1998, 1999). Although these papers present a variety of numerical results, the critical issue of the relative velocity as discussed in the preceding paragraph was not elucidated. It is interesting that the results were presented in terms of what would appear to be a relative velocity, but is, in fact, a velocity ratio.

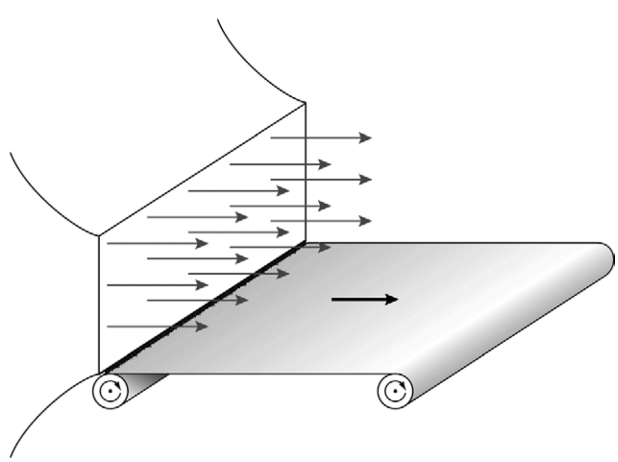

Fig. 1. Processing station consisting of a moving web situated in a parallel gas flow.

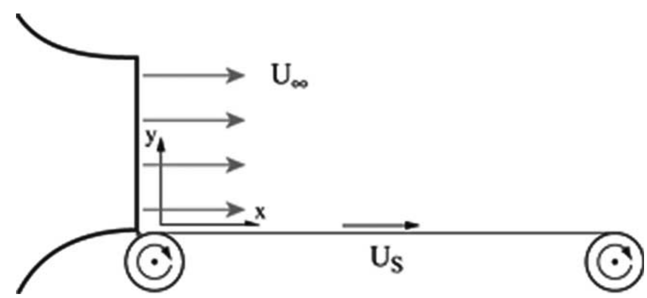

Fig. 2. Sideview of the processing station depicted in Fig. 1.

There are three foci that will be addressed in the present work. One of these is a complete numerical elucidation of the role of the relative velocity and an accounting of the errors that would be made if the relative-velocity model were used. A second focus addresses the issue of when it is permissible to neglect the slower of the participating motions without incurring a significant error. Finally, a new similarity solution based on the proper use of the relative velocity is obtained which further demonstrates the inadequacy of the relative-velocity model.

2. Drag-force results

The analysis of the fluid flow field resulting from the simultaneous motions of a moving freestream and a moving bounding surface lends itself to analysis via the method of similarity solutions (Abdelhafez, 1985). This is a very useful tool because it enables the problem,

Table 1
Listing of dimensionless drag results for the cases $U_\infty > U_S$ and $U_S > U_\infty$

$U_\infty > U_S$		$U_S > U_\infty$	
U_S/U_∞	$\frac{F_{exact}}{2\mu U_\infty W\sqrt{\frac{U_\infty L}{\nu}}}$	U_∞/U_S	$\frac{F_{exact}}{2\mu U_S W\sqrt{\frac{U_S L}{\nu}}}$
0	0.3319	0	0.4439
0.1	0.3269	0.1	0.4126
0.2	0.3132	0.2	0.3777
0.3	0.2923	0.3	0.3395
0.4	0.2651	0.4	0.2984
0.5	0.2324	0.5	0.2547
0.6	0.1946	0.6	0.2084
0.7	0.1521	0.7	0.1596
0.8	0.1053	0.8	0.1086
0.9	0.05460	0.9	0.05536

which is two-dimensional, to be reduced to one in which there is only one independent variable.

The quantity of greatest interest for practice is the drag, F, at the interface between the fluid and its bounding surface. The aforementioned similarity solutions provide *exact* results for the drag force as a dimensionless ratio which takes the following forms

$$\frac{F_{exact}}{2\mu U_\infty W\sqrt{\frac{U_\infty L}{\nu}}}, \quad U_\infty > U_S \tag{1}$$

and

$$\frac{F_{exact}}{2\mu U_S W\sqrt{\frac{U_S L}{\nu}}}, \quad U_S > U_\infty \tag{2}$$

The numerical values of these dimensionless drag results are listed in Table 1.

The tabulated results convey certain clear trends. First of all, the largest values of the dimensionless drag correspond to the situations in which either of the participating media act without motion of the other. As the magnitudes of U_∞ and U_S tend to approach each other, the values of the dimensionless drag diminish monotonically. The specific case of $U_S/U_\infty = 0$ corresponds to the Blasius flat-plate problem, and the value 0.3319 is well established in the literature. Similarly, the case of $U_\infty/U_S = 0$ represents a moving surface passing through a stationary fluid. Not only has the value 0.4439 been encountered in the previous analytical literature (Sakiadis, 1961), but it has also been verified experimentally (Tsou et al., 1967).

3. The relative-velocity model

As was discussed in Section 1, the literature treatment of situations in which both a freestream and its bounding surface are in motion in the same direction is commonly based on the relative-velocity model. *The relative-velocity model uses the magnitude of the relative velocity in conjunction with the drag formula for the case in which only one of the participating media is in motion.* For reference purposes, in order to provide information for a comparison between the correct results for the case of both media moving and those of the relative-velocity model, the latter will now be elucidated.

Two cases will be considered. The first of these, Case I, is focused on the situation in which the freestream velocity is greater than that of the moving surface, i.e., $U_\infty > U_S$. For this case, the relative velocity is

$$U_{rel}^{I} = U_\infty - U_S \tag{3}$$

To implement the relative-velocity model, the first step is to note which of the media has the higher velocity and to write the equation which represents the drag force for that medium when the other medium is not in motion. For Case I, the relevant equation is that for a stationary flat plate situated in a moving fluid. For this case, the drag force follows from Table 1 as

$$\frac{F}{2\mu U_\infty W\sqrt{\frac{U_\infty L}{\nu}}} = 0.3319 \tag{4}$$

Next, in accordance with the relative-velocity model, U_∞ is replaced by U_{rel}^{I} as defined by Eq. (3). This substitution gives

$$\frac{F^{U_{rel}^{I}}}{2\mu U_{rel}^{I} W\sqrt{\frac{U_{rel}^{I} L}{\nu}}} = 0.3319 \tag{5}$$

The second case, that for which $U_S > U_\infty$, will be termed Case II. For that case,

$$U_{rel}^{II} = U_S - U_\infty \tag{6}$$

According to the relative-velocity model, it is first appropriate to write the formula for the drag coefficient for the situation in which the plate is in motion and the fluid is stationary. That formula also follows from Table 1 as

$$\frac{F}{2\mu U_S W\sqrt{\frac{U_S L}{\nu}}} = 0.4439 \tag{7}$$

Then, upon replacing U_S by U_{rel}^{II}, there follows

$$\frac{F^{U_{rel}^{II}}}{2\mu U_{rel}^{II} W\sqrt{\frac{U_{rel}^{II} L}{\nu}}} = 0.4439 \tag{8}$$

3.1. Comparison of the exact drag results with those from the relative-velocity model

For Case I, in which $U_\infty > U_S$, it is natural to compare, in ratio form, the exact drag results, F_{exact}, from

Table 1 with $F^{U^{\mathrm{I}}_{\mathrm{rel}}}$ from the relative-velocity model as given by Eq. (5). The ratio of these drag forces is

$$\frac{F_{\mathrm{exact}}}{F^{U^{\mathrm{I}}_{\mathrm{rel}}}} = \frac{\left[\dfrac{F_{\mathrm{exact}}}{2\mu U_\infty W\sqrt{\frac{U_\infty L}{\nu}}}\right]}{0.3319}\left(\frac{U_\infty}{U^{\mathrm{I}}_{\mathrm{rel}}}\right)^{3/2}, \quad U_\infty > U_{\mathrm{S}} \tag{9}$$

The bracketed quantity in the numerator is a function of U_{S}/U_∞ and is listed in the left half of Table 1. Furthermore,

$$\left(\frac{U_\infty}{U^{\mathrm{I}}_{\mathrm{rel}}}\right)^{3/2} = \left(\frac{U_\infty}{U_\infty - U_{\mathrm{S}}}\right)^{3/2} = \left(1 - \frac{U_{\mathrm{S}}}{U_\infty}\right)^{-3/2} \tag{10}$$

Therefore, the ratio of drag forces expressed by Eq. (9) is a unique function of U_{S}/U_∞.

For Case II, a similar ratio can be formed following the same steps as were employed in deriving Eq. (9), with the result

$$\frac{F_{\mathrm{exact}}}{F^{U^{\mathrm{II}}_{\mathrm{rel}}}} = \frac{\left[\dfrac{F_{\mathrm{exact}}}{2\mu U_{\mathrm{S}} W\sqrt{\frac{U_{\mathrm{S}} L}{\nu}}}\right]}{0.4439}\left(1 - \frac{U_\infty}{U_{\mathrm{S}}}\right)^{-3/2}, \quad U_{\mathrm{S}} > U_\infty \tag{11}$$

The bracketed quantity appearing in Eq. (11) is a function of U_∞/U_{S} which is tabulated in the right half of Table 1. Therefore, the drag ratio of Eq. (11) depends only on U_∞/U_{S}.

The results of the numerical evaluations of Eqs. (9) and (11) are listed in Table 2 and shown in graphical form in Figs. 3 and 4. The deviations of the tabulated values from 1.0 are a precise measure of the errors incurred by the use of the relative-velocity model. Inspection of Fig. 3 and the first two columns of Table 2 reveals that the use of the relative velocity $U^{\mathrm{I}}_{\mathrm{rel}} = (U_\infty - U_{\mathrm{S}})$ in conjunction with the solution for a moving freestream passing over a stationary surface underestimates the drag force. The extent of the error

Table 2
Comparison of the drag forces predicted by the relative-velocity model with the exact values of the drag forces

$U_\infty > U_{\mathrm{S}}$		$U_{\mathrm{S}} > U_\infty$	
U_{S}/U_∞	$\frac{F_{\mathrm{exact}}}{F^{U^{\mathrm{I}}_{\mathrm{rel}}}}$	U_∞/U_{S}	$\frac{F_{\mathrm{exact}}}{F^{U^{\mathrm{II}}_{\mathrm{rel}}}}$
0	1.000	0	1.000
0.1	1.154	0.1	1.089
0.2	1.319	0.2	1.189
0.3	1.504	0.3	1.306
0.4	1.719	0.4	1.446
0.5	1.980	0.5	1.623
0.6	2.318	0.6	1.856
0.7	2.789	0.7	2.188
0.8	3.547	0.8	2.735
0.9	5.200	0.9	3.944

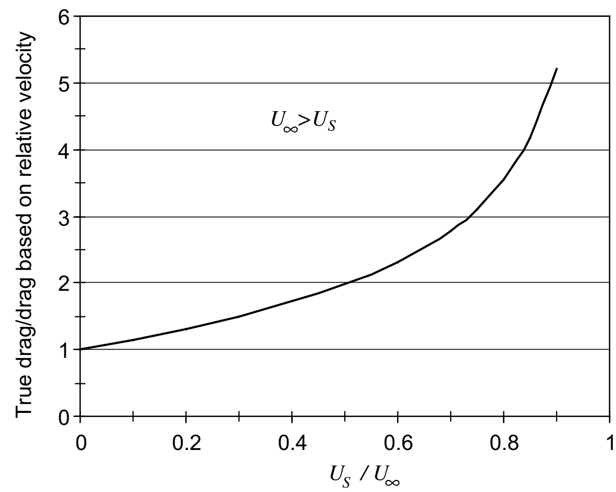

Fig. 3. Ratio of the true drag force to the drag force evaluated using the relative-velocity model. The relative-velocity model is based on the use of the relative velocity, $U^{\mathrm{I}}_{\mathrm{rel}} = (U_\infty - U_{\mathrm{S}})$ in conjunction with the drag equation, Eq. (5), which corresponds to motion of the freestream with the surface stationary.

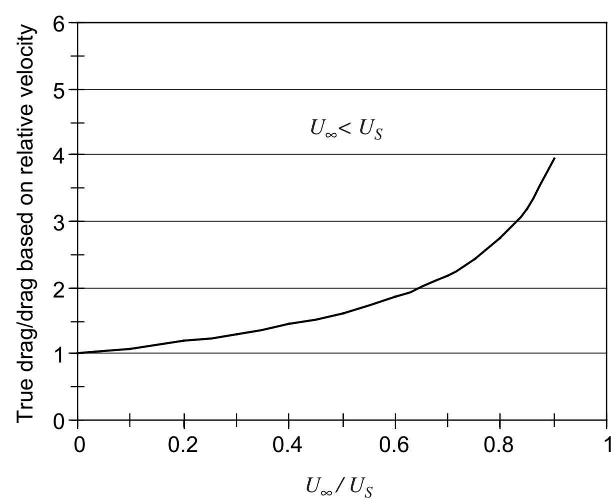

Fig. 4. Ratio of the true drag force to the drag force evaluated using the relative-velocity model. The relative-velocity model is based on the use of the relative velocity, $U^{\mathrm{II}}_{\mathrm{rel}} = (U_{\mathrm{S}} - U_\infty)$ in conjunction with the drag equation, Eq. (8), which corresponds to motion of the surface with the freestream stationary.

is magnified as the relative velocity diminishes (i.e., as U_{S}/U_∞ increases). For instance, when the surface velocity is nine-tenths of the freestream velocity, the relative-velocity model underpredicts the drag by a factor of approximately 5. For smaller ratios of the surface velocity to the freestream velocity, for example, $U_{\mathrm{S}}/U_\infty = 0.2$, the underprediction exceeds 30%.

The results for the case where $U_{\mathrm{S}} > U_\infty$ are displayed in the right half of Table 2 and in Fig. 4. The use of the relative-velocity model is seen, again, to underpredict the drag. The extent of the error is slightly smaller than that which is in evidence for the case of $U_\infty > U_{\mathrm{S}}$. For instance, when $U_\infty/U_S = 0.9$, the drag-force ratio is 3.9. When $U_S/U_\infty = 0.9$, the ratio is 5.2.

In both Case I and Case II, it is clear that the use of the relative-velocity model can lead to gross errors in the drag force.

4. Identification of the conditions under which the simultaneous motion of the participating media can be neglected

From the standpoint of engineering practice, it is often useful to know when a complicating effect can be safely neglected. For example, in cases where both the freestream and the surface are moving, it is relevant to consider when the movement of the slower of the two can be neglected. A criterion for this neglect can be formulated by seeking the conditions under which the drag force corresponding to the motion of the faster moving of the two media alone deviates by 5% from the drag force which corresponds to the simultaneous motion of the two media.

For the case in which the freestream has a velocity greater than that of the moving surface, the aforementioned criterion for the neglect of the motion of the surface is

$$\frac{\text{Drag force for freestream motion alone}}{\text{Drag force for simultaneous motion}} = 1.05 \tag{12}$$

In principle, the solution for Eq. (12) can be found from Table 1 by interpolation. However, in order to obtain an exact result, Table 1 was augmented by additional similarity solutions. The end result yields the following solution for Eq. (12):

$$U_S/U_\infty \leqslant 0.183, \quad \text{for neglecting the motion of the surface.} \tag{13}$$

The fact that the motion of the surface can be neglected for U_S/U_∞ as large as 0.183 is of practical importance and suggests a simplified approach for these cases.

A similar analysis can be carried out for the situation in which the surface is moving much faster than the freestream. Under that condition, the freestream motion can be neglected when

$$U_\infty/U_S \leqslant 0.0686, \quad \text{for neglecting the freestream motion} \tag{14}$$

In contrast to the finding for the case in which the dominant motion is that of the freestream, the case of surface-dominated motion is quite sensitive to the motion of the freestream.

5. Similarity solutions based on the relative velocity between the participating media

Thus far, when the relative velocity between the moving surfaces has been considered, it has always been in connection with the *relative-velocity model*. As has been pointed out, that model is logically incorrect because it utilizes, as its basis, solutions for the case in which only one of the participating media is in motion. A distinctly different relative-velocity model will be derived which will, in fact, be an exact similarity solution of the governing equations for boundary layer flow. The new model will be termed *the similarity-based, relative-velocity model.*

To begin the derivation of the similarity model, it is useful to define

$$\xi = y\sqrt{\frac{|U_{\text{rel}}|}{\nu x}} \tag{15}$$

and

$$\psi = \sqrt{|U_{\text{rel}}|\,\nu x}\,f(\xi) \tag{16}$$

where $|U_{\text{rel}}| = |U_\infty - U_S|$. These variables can be employed to transform the conservation equations to similarity form

$$\frac{d^3 f}{d\xi^3} + \frac{1}{2}\frac{d^2 f}{d\xi^2}\frac{df}{d\xi} = 0 \tag{17}$$

The fact that this equation is an ordinary differential equation is testimony to the fact that the magnitude of the relative velocity is a suitable parameter for the similarity transformation.

It still remains to demonstrate that the boundary conditions depend only on ξ and are independent of x. To this end, the velocity components are needed. They are expressible in terms of f and $df/d\xi$ as

$$u = |U_{\text{rel}}|\frac{df}{d\xi} \tag{18}$$

$$v = \frac{1}{2}\sqrt{\frac{\nu|U_{\text{rel}}|}{x}}\left(\xi\frac{df}{d\xi} - f\right) \tag{19}$$

The physical boundary conditions that are to be applied to Eqs. (18) and (19) are

$$v = 0 \quad \text{and} \quad u = U_S \text{ at } y = 0 \tag{20}$$

$$u \to U_\infty \text{ as } y \to \infty \tag{21}$$

The end result of the application of the boundary conditions is

$$f(0) = 0, \quad \frac{df}{d\xi}(0) = \frac{U_S}{|U_{\text{rel}}|}, \quad \text{and} \quad \frac{df}{d\xi}(\xi \to \infty) = \frac{U_\infty}{|U_{\text{rel}}|} \tag{22}$$

The transformed boundary conditions conveyed by Eq. (22) are seen to be constants, independent of x. Therefore, the similarity transformation is complete.

From the standpoint of calculation, it is necessary to prescribe the values of df/dξ at $\xi = 0$ and as $\xi \to \infty$. To this end, it is convenient to define

$$\alpha = \frac{U_\infty}{U_S} \tag{23}$$

so that

$$\frac{df}{d\xi}(0) = \frac{U_S}{|U_{rel}|} = \frac{1}{\left|1 - \frac{U_\infty}{U_S}\right|} = \frac{1}{|1-\alpha|} \tag{24}$$

and

$$\frac{df}{d\xi}(\xi \to \infty) = \frac{U_\infty}{|U_{rel}|} = \frac{\frac{U_\infty}{U_S}}{\left|1 - \frac{U_\infty}{U_S}\right|} = \frac{\alpha}{|1-\alpha|} \tag{25}$$

A review of Eqs. (17), (22), (24), and (25) indicates a complete definition of the *similarity-based*, *relative-velocity model*. To implement the numerical solutions, values of $\alpha = U_\infty/U_S$ were parametrically assigned from 0 (stationary fluid, moving surface) to ∞ (stationary surface, moving fluid). The values of $\left|\frac{d^2f}{d\xi^2}(0)\right|$ that correspond to the specific values of α are listed in Table 3. These tabulated values can be used to evaluate the drag from

$$F_{exact} = 2\mu|U_{rel}|W\sqrt{\frac{|U_{rel}|L}{\nu}}\left|\frac{d^2f}{d\xi^2}(0)\right| \tag{26}$$

The numerical values of the drag force from Eq. (26) have to match the numerical values of the drag force in Table 1. It was verified that this matching was indeed achieved to four significant figures. This matching also provides relationships between the tabulated values conveyed in Tables 1 and 3. Those relationships are

$$\frac{F_{exact}}{2\mu U_\infty W\sqrt{\frac{U_\infty L}{\nu}}} = \left|1 - \frac{1}{\alpha}\right|^{3/2} \frac{F_{exact}}{2\mu|U_{rel}|W\sqrt{\frac{|U_{rel}|L}{\nu}}}, \quad U_\infty > U_S \tag{27}$$

and

$$\frac{F_{exact}}{2\mu U_S W\sqrt{\frac{U_S L}{\nu}}} = |\alpha - 1|^{3/2} \frac{F_{exact}}{2\mu|U_{rel}|W\sqrt{\frac{|U_{rel}|L}{\nu}}}, \quad U_S > U_\infty \tag{28}$$

Table 3
Listing of $\left|\frac{d^2f}{d\xi^2}(0)\right|$ values for the *similarity-based*, *relative-velocity model*

$\alpha = \frac{U_\infty}{U_S}$	$\frac{F_{exact}}{2\mu\|U_{rel}\|W\sqrt{\frac{\|U_{rel}\|L}{\nu}}} = \left\|\frac{d^2f}{d\xi^2}(0)\right\|$
0.000	0.4439
0.1000	0.4832
0.2000	0.5279
0.3000	0.5797
0.4000	0.6421
0.5000	0.7204
0.6000	0.8238
0.7000	0.9713
0.8000	1.214
0.9000	1.717
1.111	1.726
1.250	1.177
1.429	0.9257
1.667	0.7689
2.000	0.6573
2.500	0.5704
3.333	0.4991
5.000	0.4377
10.00	0.3829
∞	0.3319

6. Concluding remarks

The focus of this investigation is fluid flows which occur when both a freestream flow and its bounding surface are in motion in the same direction. All combinations of velocity magnitudes of the participating media were considered for numerical evaluation. It was found that when the velocity, U_∞, of the moving freestream was greater than the velocity, U_S, of the bounding surface, the drag force was virtually unaffected by the motion of the surface when $U_S/U_\infty < 0.183$. On the other hand, for the case in which $U_S > U_\infty$, the drag force is affected by the moving freestream for $U_\infty > 0.0686U_S$. These findings illustrate that the dominant flows have different sensitivities to the presence of the lesser flows for the two generic cases. These results are part of a broader picture which indicates an asymmetry between the cases $U_\infty > U_S$ and $U_S > U_\infty$.

A second focus of this paper was to investigate and clarify the role of the relative velocity in situations where both the fluid and the surface are in motion. From a study of the literature (Bejan, 1993; Cengel, 2003; Incropera and DeWitt, 2002; Kaviany, 2002; Kreith and Bohn, 2001; Rohsenow and Choi, 1961; Suryanarayana, 1995), a model based on the relative velocity of the two media was identified. This model may be termed the *relative-velocity model.* According to this model, the relative velocity is used in conjunction with the drag-force formula for the dominant of the two flows acting alone. It was demonstrated that the use of the relative-velocity model underpredicts the drag. The extent of the error increases as the two participating velocities approach each other in magnitude. For those situations, errors as much as a factor of five were encountered. On this basis, it is recommended that the relative-velocity model not be employed for practical calculations.

The use of the relative velocity as the basis of exact similarity solutions of the boundary layer equations was demonstrated to be valid. This model is termed the *similarity-based, relative-velocity model*. It is altogether distinct from the relative-velocity model of the literature. By the use of the similarity-based, relative-velocity model, exact solutions for the drag were obtained for the entire range of operating conditions bounded by freestream motion over a stationary surface and by a moving surface bounding a stationary fluid.

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